Summer Solstice 2018 Conference on Discrete Models of Complex Systems
Gdańsk, June 25-27, 2018

On the reversibility problem of finite hexagonal cellular automata over Z3

Adam Dzedzej, *Institute of Mathematics, Faculty of Mathematics, Physics and Informatics, University of Gdańsk, 80-308 Gdańsk, Poland*
Barbara Wolnik, *Institute of Mathematics, Faculty of Mathematics, Physics and Informatics, University of Gdańsk, 80-308 Gdańsk, Poland*
Antoni Augustynowicz, *Institute of Mathematics, Faculty of Mathematics, Physics and Informatics, University of Gdańsk, 80-308 Gdańsk, Poland*
Maciej Dziemiańczuk, *Institute of Informatics, Faculty of Mathematics, Physics and Informatics, University of Gdańsk*
Jan M. Baetens, *KERMIT, Department of Data Analysis and Mathematical Modelling, Ghent University* Bernard De Baets, *KERMIT, Department of Data Analysis and Mathematical Modelling, Ghent University*

Reversible cellular automata (CAs) are of particular interest in CA applications since they can be used to simulate, for instance, gas or fluid dynamics. Unfortunately, determining whether a CA is reversible is an intricate task, especially in two or more dimensions. Moreover, in general, reversibility is not decidable [1]. However, in the case of CAs with the state set Z3 on two-dimensional hexagonal grids (HCAs), the problem of reversibility has been regarded as completely resolved. For null boundary conditions Siap et al. [2] claimed to have classified the HCAs and sizes of grids on which they are reversible. It appears that the results presented in [2] are not valid. The problem of reversibility of HCAs with periodic boundary conditions was studied in [3]. Nevertheless, the tools introduced there do not allow to decide whether a given HCA is reversible or not. Additionally, they apply to a restricted list of HCAs by imposing conditions on local rule coefficients.

We give counterexamples to theorems stated in [2] and present a correct and extended classification of HCAs over Z3 with periodic boundary conditions.

References:

[1] Kari, J.: Reversibility of 2D cellular automata is undecidable. Physica D: Nonlinear Phenomena 45(1), 379-385 (1990)

[2] Siap, I., Akin, H., Uguz, S.: Structure and reversibility of 2D hexagonal cellular automata. Computers and Mathematics with Applications 62(11), 4161-4169 (2011)

[3] Uguz, S., Siap, I., Akin, H.: 2-dimensional reversible hexagonal cellular automata with periodic boundary. Acta Physica Polonica A 123(2), 480-483 (2013)