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Solving the relaxed density classification problem by means of two-dimensional Affine Continuous Cellular Automata.

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We consider particular two-dimensional cellular automata (CAs) for solving the density classification problem (DCP) in some generalized sense. It is said that a d-dimensional binary CA solves the DCP if it evolves to the homogeneous state (all 0s or all 1s) that corresponds to the majority state of the initial configuration. The first formulation of the DCP concerns one-dimensional binary CAs. It was proved for any dimension that there exists no d-dimensional binary CA that solves the DCP. The proof follows from the fact that one of the necessary conditions for a binary CA to classify density for all initial configurations (of any size) is that it must be density-conserving, conflicting the goal of the DCP.

To overcome this hurdle, researchers have modified the original DCP by allowing two CAs instead of a single, adding memory, using non-deterministic CAs or by defining a different output specification. In our previous work we considered Continuous CAs (CCAs), i.e. CAs with real-valued states, which are also known as coupled-map lattices and look for an infallible classifier for one dimension. We focused on one of the simplest families of CCAs, so-called Affine CCAs (ACCAs), in which the local rule is affine in each variable and the values are in the range [0,1]. In this family there is no rule that solves the classical DCP, but by a slight relaxation of the output specification, we found an entire family of density-conserving ACCAs solving this relaxed DCP. Our simulations showed that for such ACCAs with a range one neighbourhood the best performing ACCA, acts as the traffic rule that is mixed with the identity rule.

Now we consider two-dimensional ACCAs, which implies a very high computational complexity. We examine which two-dimensional density-conserving ACCAs are performing the best. The answer to this question is interesting because in two dimensions there is no rule that solves the spacing problem like the traffic rule does for one dimension.